Pilot Modeling and Closed-Loop Analysis of Flexible Aircraft in the Pitch Tracking Task

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This paper addresses the appropriate modeling techniques for pilot vehicle analysis of large flexible aircraft when the frequency separation between the rigid-body modes and the dynamic aeroelastic modes is reduced. This situation was shown to have significant effects on pitch-tracking performance and subjective rating of the task, obtained via fixed-base simulation. Further, the dynamics in these cases are not well modeled with a rigid-body-like model obtained by including only "static elastic" effects, for example. It is shown that pilot/vehicle analysis of this data supports the hypothesis that an appropriate pilot-model structure is an optimal-control pilot model of full order. This is in contrast to the contention that a representative model is of reduced order when the subject is controlling high-order dynamics as in a flexible vehicle. The key appears to be in the correct assessment of the pilot's objective of attempting to control "rigid body" vehicle response, a response that must be estimated by the pilot from observations contaminated by aeroelastic dynamics. Finally, a model-based metric is shown to correlate well with the pilot's subjective ratings.

Nomenclature

A	= system matrix in state model
B	= control input matrix in state model
C_p	= pilot observation matrix for determining pilot model
D	= disurbances input matrix in state model
E	= expected value operator
e_{dis}	= displayed tracking error, $\theta_{dis} - \theta_c$
$F_{(\cdot)}$	= aeroelastic mode generalized force due to parameter $(\cdot)$
$G(s)$	= transfer function for aeroelastic vehicle $\theta_{dis}(s)/\delta_p(s)$ response
$\bar{g}_x$	= pilot model control gain vector
$H_{dis}(s)$	= human pilot describing function with displayed attitude tracking objective
$H_R(s)$	= human pilot describing function with rigid attitude tracking objective
J_p	= pilot model quadratic objective function
ℓn	= natural logarithm
$M_{(\cdot)}$	= aircraft aerodynamic moment stability derivative associated with parameter $(\cdot)$
Q	= weighting matrix in J_p reflecting task objective
q	= rigid body pitch rate
r	= pilot's control weighting in J_p, usually adjusted to yield an appropriate τ_n
$T_{(\cdot)}$	= observation threshold on observed variable $(\cdot)$ in pilot model
V_m, V_y	= covariance intensity matrices for v_m and $\bar{v}_y$ in pilot model
$v_m, \bar{v}_y$	= random "motor" noise and observation noise vector in pilot model
w	= "white" random disturbance process
$\bar{x}$	= state vector
$\hat{x}$	= state estimate vector
$\bar{y}_p$	= vector of pilot-observed variables
y_i	= elements of $\bar{y}_p$
$Z_{(\cdot)}$	= aircraft aerodynamic force stability derivative associated with parameter $(\cdot)$
$\bar{z}_p$	= vector of controlled responses
α	= aircraft aerodynamic angle of attack
$\delta(t)$	= impulse (delta function)
δ_p	= pilot's control input (e.g., elevator deflection)
ζ	= modal damping
η_i	= generalized normal coordinate for the ith elastic mode
θ_c	= commanded attitude angle
θ_{dis}	= displayed attitude angle including elastic deformation $(\theta_R + \theta_E)$
θ_E	= elastic contribution to displayed attitude angle
θ_R	= rigid body attitude angle
σ	= standard deviation
τ	= pilot's observation time delay
τ_n	= pilot's neuromotor time constant
$\phi_\theta{}^i$	= elastic mode slope at cockpit location for mode i
ω	= undamped natural frequency of an oscillatory mode

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Introduction

WITH the advent of larger aircraft in the future, and the desire for even lighter structures for fuel economy, the potential exists for significant contributions of the aeroelastic modes of these vehicles in the responses to pilot input and/or atmospheric turbulence. To develop handling qualities specifications or synthesize flight control laws and cockpit displays, it is therefore important to understand the pilot-vehicle interactions in such cases. The fundamental work of Swaim and Yen[1] is significant in this regard. Pilot-vehicle analysis techniques appropriate for vehicles with higher-order dynamics due to aeroelastic effects (or higher order control system dynamics, for that matter) are required for the continued development of this understanding.

When sufficient frequency separates the elastic modes from the rigid-body modes, truncating or residualizing (correcting for "static elastic" effects on the aerodynamics) yields an acceptable vehicle model, or an "equivalent" rigid-vehicle model. And, of course, in such cases conventional pilot-vehicle analyses based on the crossover model[2] or the optimal control model (OCM)[3] are appropriate. A fundamental issue arises, however, when this frequency separation is reduced—the situation for large, flexible aircraft. The question of what modeling and analysis approaches are valid in this case is now germane.

A specific question is raised, for example, as to the level of complexity, or the dynamic order of the pilot's dynamics, based on his "internal model" of the system he is controlling. Proponents of the lower-order modeling hypothesis argue that a full-order optimal control model, for example, yields predicted pilot-vehicle performance that is overly optimistic. Gerlach[4] states in his biomorphic pilot model that the dynamic order of the internal model is not greater than two, regardless of the order of the plant. This statement was not, however, substantiated by experimental data. Swaim and Poopaka[5] hypothesized an internal model of order four, the same order as a rigid-body model of the aircraft longitudinal dynamics. Their results were not compared to experimental findings either, though their quantitative trends appear promising. Still other "reduced-order" models are discussed in Ref. 6.

It is clear that in many cases of pilot-vehicle analysis simpler or low-order models yields adequate results and may be easier to manipulate. This should not be taken as proof that a "high-order" model is invalid. In fact, it will be shown that a pilot-vehicle analysis of experimental pitch-tracking data utilizing a full-order optimal control model leads to very good analytical/experimental correlation. Such results clearly support a full-order modeling hypothesis. As will be shown, the key would appear to be the appropriate modeling of the pilot's objective in the task, again underscoring the importance of this aspect of the modeling.

Finally, a model-based rating metric is proposed that is shown to correlate well with the pilot's subjective rating of the vehicle in the task, thus increasing the utility of the modeling approach.

Experimental Data

The data to be considered were obtained via fixed-base simulation of a pursuit attitude pitch tracking task, using a family of vehicle dynamic models.[7] The baseline model was representative of a vehicle as shown in Fig. 1, flying at sea level at a Mach number of 0.85. The other models are parametric variations of the baseline, obtained by decreasing the in-vacuo vibration frequency of the first and/or second fuselage bending mode. The mode shapes were assumed constant and aeroelastic vehicle models derived for each case. (One might consider all these configuations then as having the geometry as in Fig. 1, but the material properties of the vehicle are changed in such a way so as to result in the different structural vibration frequencies.) A summary of seven of the configurations, listed in terms of the vibration modal frequencies assumed and the resulting longitudinal eigenvalues, is given in Table 1. Complete modal analyses of these vehicles are presented in Ref. 7.

Experimental tracking data were obtained for all configurations with four subjects (pilots). The displays included a fixed reference, a command bar, and an attitude bar, all displayed on a CRT. The vertical position of the command bar indicated the "commanded" pitch attitude θ_c, while the vertical position of the attitude bar (above or below the reference) indicated the vehicle attitude, including the elastic slope at the cockpit relative to the rigid body vehicle axis. This displayed attitude θ_{dis} would be the inertial attitude measured by a sensor at the pilot station, and is related to the rigid body attitude θ_R by

$$\theta_{\text{dis}}(t) = \theta_R(t) - \sum_{i=1}^{n} \phi_\theta^i \eta_i(t)$$

where ϕ_θ^i in the elastic mode slope of mode i at the cockpit, η_i, is the generalized coordinate associated with elastic with elastic mode i, and n is the number of elastic modes in the model. The geometric relation between these variables is shown in Fig. 2. It is important to note that θ_R may be thought of as the vehicle attitude and that the displayed θ_{dis} is an indication of θ_R, but is contaminated with the motion associated with the vehicle aeroelastic modes.

Sample time histories obtained for subjects attempting to track a slowly varying (≈ 5 deg/s) commanded attitude θ_c are shown for configurations 1, 3, 4, and 7 in Figs. 3-6. Note the characteristically different attitude histories, although configurations 3 and 4 included an aeroelastic mode (and two modes in configuration 7) that had a natural frequency more than three times greater than the configuration's short-period frequency. It should be clear from these time histories that the attitude dynamics of these vehicles cannot be approximated with any rigid-body-like model.

Listed in Table 2 are subjects' comments corresponding to these seven configurations, along with the statistics on tracking error and Cooper-Harper[8] subjective ratings. Note that the subjects' comments indicate that the presence of the oscillations were perceived in all cases except configuration 1, and that in the case of configurations 3 and 7, the dynamics were completely unacceptable.

Pilot-Vehicle Analysis

A pilot-vehicle analysis is presented here, utilizing a full-order, optimal-control model (OCM) of the human operator.[3] The hypothesis of this model is that the well trained, well motivated subject (pilot) chooses his control input $\delta_p(t)$ subject to human limitations, so as to minimize his "cost function" in the task. Furthermore, this cost function is assumed to be reasonably approximated by a quadratic function of the form

$$J_p = E\left\{\lim_{T\to\infty} \frac{1}{T}\int_0^T \bar{z}_p^T Q \bar{z}_p + r\dot{\delta}_p^2)\,dt\right\}$$

with $\bar{z}_p$ the vector of responses the pilot is attempting to control. The dynamics being controlled are expressed in the familiar form

$$\dot{\bar{x}} = A\bar{x} + B\delta_p + Dw; \qquad E\{w(t)w(\sigma)\} = W\delta(t-\sigma)$$

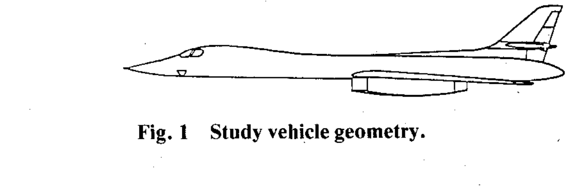

Fig. 1 Study vehicle geometry.

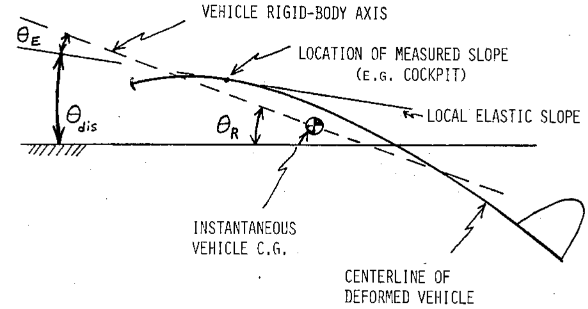

Fig. 2 Geometric relationships.

Table 1 Configuration summary

Configuration	In-vacuo frequencies Mode 1	Mode 2	Phugoid mode[a]	Short period mode[a]	Elastic modes First[a]	Second[a]
1 (baseline)	13.7	21.2	(0.02,0.08)	(0.53,2.8)	(0.05,13.3)	(0.02,21.4)
2	9.2	21.2	(0.0 0.06)	(0.52,2.6)	(0.09,8.8)	(0.02,21.4)
3	6.2	21.2	(+0.09)(−0.08)	(0.52,1.8)	(0.20,5.9)	(0.02,21.4)
4	13.7	4.8	(+0.15)(−0.13)	(0.69,1.6)	(0.05,13,3)	(0.11,6.0)
5	10.7	9.3	(0.05,0.06)	(0.55,2.4)	(0.11,10.3)	(0.0,9.8)
6	11.7	11.7	(0.0, 0.05)	(0.54,2.6)	(0.08,11.7)	(0.0, 11.6)
7	6.9	6.9	(+0.18)(−0.15)	(0.70,1.4)	(0.19, 7.3)	(0.0,6.9)

[a] Modal parameter notation. Complex (ξ,ω), Real $(-P)$. All frequencies in rad/s.

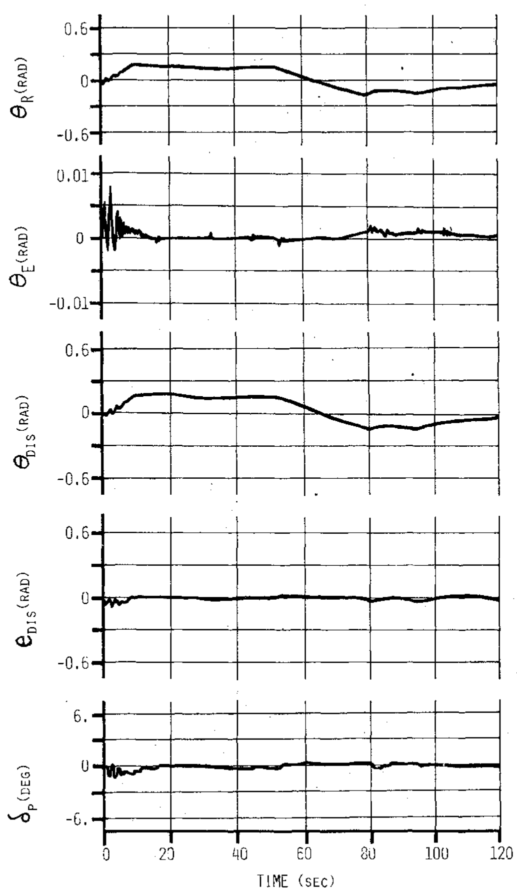

Fig. 3 Time histories, configuration 1.

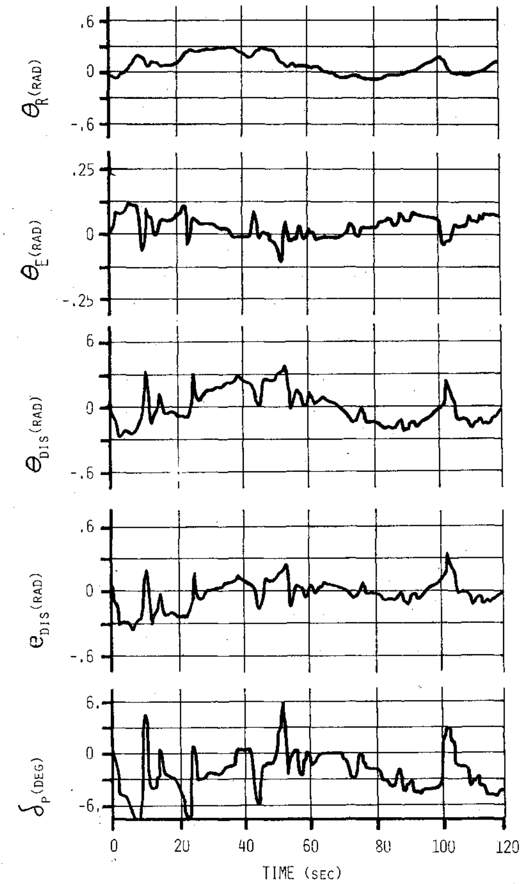

Fig. 4 Time histories, configuration 3.

with the pilot's observation vector $\bar{y}_p$ expressed as a linear combination of delayed states, and contaminated with observation noise, or

$$\bar{y}_p = C_p \bar{x}(t-\tau) + \bar{v}_y; \qquad E\{v_y(t)v_y^T(\sigma)\} = V_y\delta(t-\sigma)$$

where τ is the pilot's observation time delay. The control law considered to mimic the dynamic interaction between the pilot and vehicle is then taken as

$$\tau_n \dot{\delta}_p = \bar{g}_x \hat{\bar{x}} - \delta_p + v_m \; ; \quad E\{v_m(t)v_m^T(\sigma)\} = V_m\delta(t-\sigma)$$

where $\bar{g}_x$ are contol gains determined from the minimization of J_p. The state estimate $\hat{\bar{x}}$ is obtained from a Kalman filter, cascaded with a least-mean-square predictor that arises due to the presence of the human operator's observation delay τ. A block diagram of this modeling structure is shown in Fig. 7.

Values for τ, τ_n, V_m, and V_y are based on empirical man-machine data. "Rules" for adjusting them to be consistent with the task are continuously being developed through experimental research. However, a fairly "standard" set[3] found to agree with a wide range of vehicle dynamics and laboratory tracking tasks is given in Table 3. These values will be used throughout this analysis.

The key modeling parameter to be discussed now is the objective function. In the case of laboratory tracking tasks involving simple plants (K/s, K/s^2, *etc.*), selecting z_p to be simply the scalar tracking error $(\theta_c - \theta_{dis})$ perceived on the display is usually a good definition of the task (i.e., minimization of displayed error). This simple expression has also been found to be most appropriate in analysis of the attitude tracking task in aircraft in-flight simulation experiments.[9] Such a selection for z_p is consistent with the idealized closed-loop representation in Fig. 8, identical to the structure assumed in the crossover model.[2] As shown in Ref. 10, in many cases one can use the frequency or the time domain (OCM) to estimate the pilot's dynamic behavior, or $H_{dis}(s)$ in Fig. 8. In either case, the implicit or explicit assumption is that the pilot is attempting to minimize displayed error $e_{dis} = (\theta_c - \theta_{dis})$ and the resulting $H_{dis}(s)$ reflects this.

Table 2 Summary of experimental results

Configuration	RMS error. deg (mean $\pm 1\sigma$)	Cooper-Harper rating (mean $\pm 1\sigma$)	Comments
1	1.2 ± 0.6	1.6 ± 0.4	Very nice; no problem
2	1.0 ± 0.5	2.0 ± 0.3	Little oscillation; more difficult than C1; slight control response lag.
3	5.7 ± 1.1	5.9 ± 1.9	Difficult; required high concentration; PIO problem; extreme response lag.
4	1.9 ± 0.3	3.1 ± 1.1	Little more difficult than C1; slightly sluggish attitude response.
5	1.2 ± 0.5	1.9 ± 0.4	Not difficult, little more oscillation, but could ignore it and fly rigid body; like config. 2.
6	1.5 ± 0.7	2.0 ± 0.5	Pretty good; same as 2.
7	7.6 ± 2.8	6.7 ± 1.6	With severe oscillations, virtually uncontrollable. Abrupt control inputs led to disaster.

Table 3 Pilot model parameters

Observation vector	$\hat{y}_p^T [e_{\rm dis}, \dot{e}_{\rm dis}, \theta_{\rm dis}, \dot{\theta}_{\rm dis}]$ $e_{\rm dis} = \theta_c - \theta_{\rm dis}$
Fractional attention allocation	0.25 on y_i, $i = 1 - 4$
Full attention observation noise ratio	-20 dB (all observed variations)
Observation thresholds (angles at the pilot's eye)	$T_\theta = 0.05$ deg $T_{\dot{\theta}} = 0.10$ deg/s
Observation delay	$\tau = 0.2$ s
Neuromuscular lag	$\tau_n \cong 0.1$ s
Neuromotor noise ratio	-20 dB

It is at this point that the issue arises as to the appropriate complexity of $H_{\rm dis}(s)$ (or the complexity of the pilot's internal model) when the dynamics he is controlling $G(s)$ is of higher dynamic order than a rigid vehicle. It will now be shown that the experimental results previously presented support the full-order hypothesis *with the proper statement of the pilot's objective.*

Comments from the subjects (Table 2), for example, as well as Swaim and Poopaka[5] indicate that the subjects are attempting to control rigid body (vehicle) attitude error, rather than displayed error. That is, their control strategy lies in attempting to minimize $(\theta_c - \theta_R)$ rather than $(\theta_c - \theta_{\rm dis})$. The appropriate idealized representation is as shown in Fig. 9, rather than Fig. 8. The pilot's dynamic model is then represented as $H_R(s)$ rather than $H_{\rm dis}(s)$.

For reference, the transfer functions in Figs. 8 and 9 may be considered obtainable from relations of the form

$$\begin{bmatrix} (s - Z_\alpha) & -(1 + Z_q) & -(Z_{\dot{A}} s + Z_A) \\ -M_{\dot{\alpha}} s + M_\alpha) & (s - M_q) & -(M_{\dot{A}} s + M_A) \\ -F_\alpha & -F_q & (s^2 + 2\zeta\omega_A s + \omega_A^2) \end{bmatrix} \times \begin{bmatrix} \alpha/\delta_p \\ \dot{\theta}_R/\delta_p \\ \theta_E/\delta_p \end{bmatrix} = \begin{bmatrix} Z_\delta \\ M_\delta \\ F_\delta \end{bmatrix}$$

where in the above equation the velocity perturbation has been ignored (the short period approximation) and only one aeroleastic mode is included for simplicity.

To test this hypothesis, a pilot vehicle analysis of the experimental data was performed using the cost function

$$J_p(\theta_R) = E\left\{\lim_{T\to\infty} \frac{1}{T} \int_0^T [(\theta_R - \theta_c)^2 + r\dot{\delta}_p^2]\,dt\right\}$$

to determine the control law and pilot model. The seven configurations above were analytically evaluated using the above cost function and the tracking command modeled as in Ref. 9., or

$$\ddot{\theta}_c + 0.5\dot{\theta}_c + 0.25\theta_c = w(t)$$

with the intensity of the driving noise $w(t)$ selected to yield $\sigma_{\theta_C} = 7.9$ deg. The results are shown in Fig. 10, in which rms displayed error $(\theta_{\rm dis} - \theta_c)$ for the model is compared with experiment. Good agreement was obtained in all cases, with the analytical results clearly "exposing" the poor configurations.

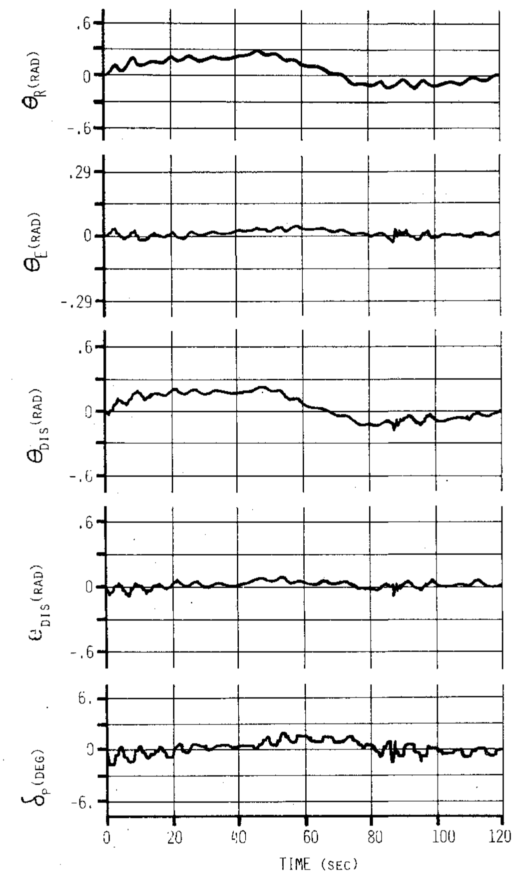

Fig. 5 Time histories, configuration 4.

This trend is in contrast to the cases[5] in which a full-order model predicts tracking performance overly optimistic because of the use of an inappropriate objective function. For example, re-evaluating configuration 3 with an assumed objective function reflecting a desire to minimize displayed error, or choosing

$$J_p(\theta_{\rm dis}) = E\left\{\lim_{T\to\infty} \frac{1}{T} \int_0^T [(\theta_{\rm dis} - \theta_c)^2 + r\dot{\delta}_p^2]\,dt\right\}$$

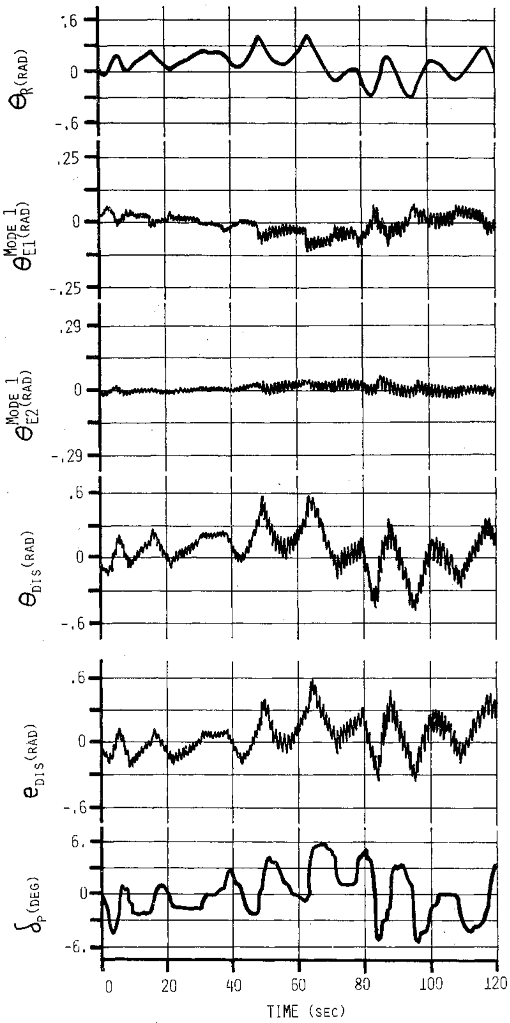

Fig. 6 Time histories, configuration 7.

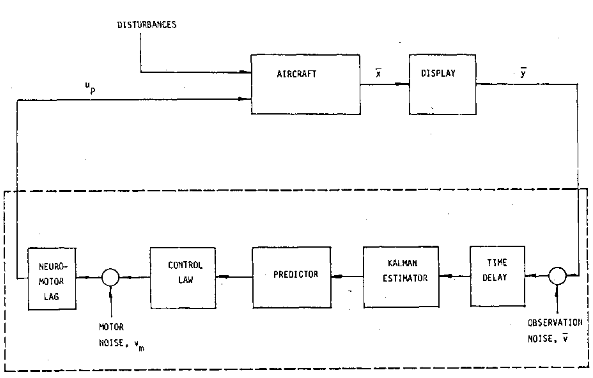

Fig. 7 Optimal control model structure.

yields the analytically determined displayed error of 1.8 deg, also shown in Fig. 10.

It is these optimistic performance predictions that have led to conjecture on the validity of a full-order model. The hypothesis is that the order of the internal model is a human limitation that must be reflected in the model structure. On the other hand, the recognition that displayed error may not be the parameter being minimized casts the problem as one of objective rather than of limitation.

The significantly degraded tracking performance is explained by the fact that the response being controlled, or the rigid body vehicle attitude, must be estimated using observations of θ_{dis} that are contaminated by the elastic mode response. As the elastic mode frequencies approach those of the predominately rigid-body modes, the ability to estimate the vehicle's rigid-body attitude diminishes. It is interesting to note, furthermore, that this is a natural characteristic of a Kalman filter, even if it is of full order. Therefore, the OCM structure needs no modification to include this property.

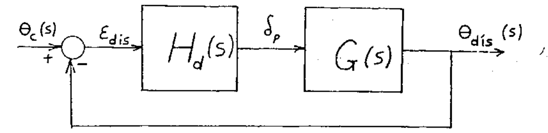

Fig. 8 Conventional idealized representation.

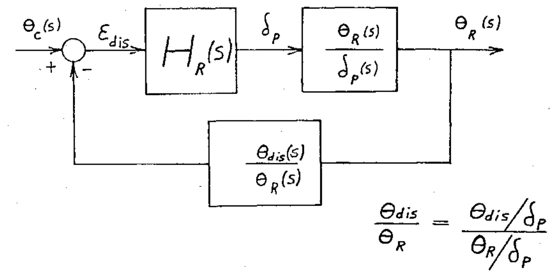

Fig. 9 Idealized representation with flexibility.

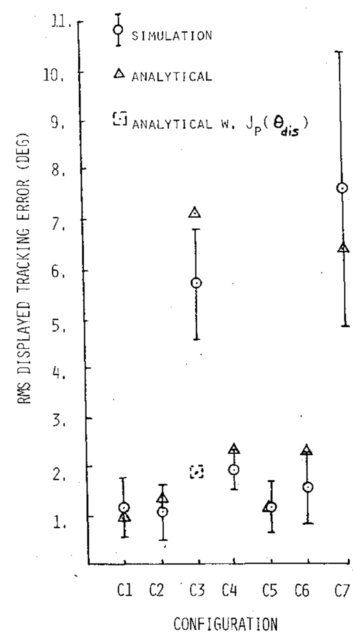

Fig. 10 Tracking error results.

As an example demonstrating this fact, consider the results shown in Fig. 11. Plotted here is the rms error in the Kalman filter estimate of the rigid-body attitude angle θ_R and the elastic mode contribution θ_E, as a function of the in-vacuo frequency of the first fuselage bending model. These errors are an indication of the ability of the human subject to separate the rigid body and elastic responses in the displayed θ_{dis} (see Table 2). Note that for baseline Configuration one the estimation error on θ_R is about 0.5 deg, an error more than doubled when the in-vacuo bending frequency of the elastic mode is less than 9 rad/s. This trend, furthermore, is magnified due to the human subject's perception error characteristics (modeled in terms of the noise intensities V_y and V_m), which lead to a constant noise-to-signal ratio. Note that this fact is a magnification factor only. The requirement for estimation of θ_R is the fundamental cause of the problem.

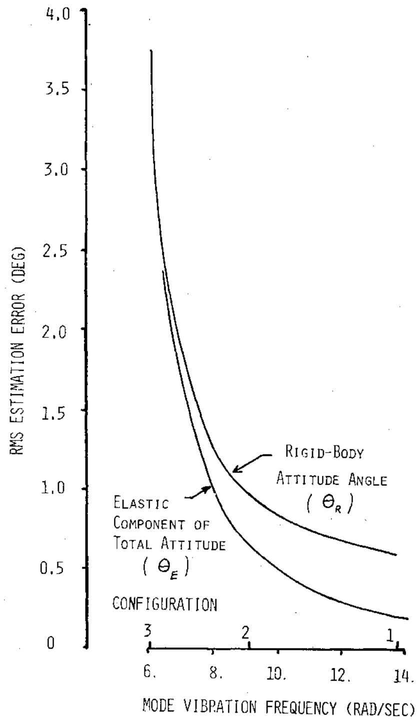

Fig. 11 Estimation errors of attitude angles.

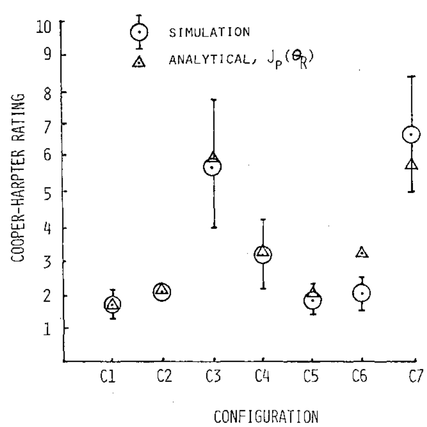

Fig. 12 Subjective rating results.

Finally, consider the results shown in Fig. 12, or the Cooper-Harper subjective rating of the task. We will here invoke the rating hypothesis of Hess,[10] which states that properly modeled, the magnitude of the cost function J_p is correlated with the subjective rating of the pilot in the task. Based on an empirically obtained metric[11,12] for the relative rating between two configurations (denoted *a* and *b*)

$$\Delta \text{Rating}_{a-b} = 2.5\, n(J_{P_a}/J_{P_b})$$

we obtain the model-based ratings given in the future. (Configuration 1 is the baseline for rating calibration.) Again, the agreement is quite good, with the "analytical" results clearly exposing the bad configurations.

Summary and Conclusions

The experimental data presented and reviewed here clearly indicates that dynamic aeroelastic effects can significantly affect attitude tracking performance and subjective rating. Furthermore, truncation or residualization of these modes to obtain a reduced-order rigid body model should not be expected to capture these important dynamic characteristics properly. The significant dynamics must be included in the model and their effects on the vehicle handling qualities assessed.

Pilot-vehicle analysis of these experimental results supports the hypothesis of a full-order pilot model being appropriate. However, the careful assessment of the task objective is a key requirement. A complex objective function is not required. The "correct" one appears to be simple in form. The pilot would appear to choose his control input so as to attempt to minimize vehicle rigid body attitude error, a response that must be estimated from perception of the indicated attitude contaminated by aeroelastic dynamic response.

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